

Primordial black holes, zero-point energy and CMB: The cosmic connection

S. Zeynizadeh^{1,2,*} and M. Nouri-Zonoz^{3,†}

¹*Department of Physics, Sharif University of Technology P.O. Box 11365-9161, Tehran, Iran*

²*School of physics, Institute for Research in Fundamental Sciences (IPM) P.O. Box 19395-5531, Tehran, Iran*

³*Department of Physics, University of Tehran, North Karegar Ave., P.O. Box 14395-547, Tehran, Iran.*

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We propose a possible resolution to the cosmological constant problem through a scenario in which the universe is composed of three components: matter, radiation (CMB) and vacuum energy such that vacuum energy is not constant and is decaying into the matter component. Matter in this scenario consists of baryonic matter and primordial black holes (PBHs) as the dark matter. Local equilibrium condition between PBHs and CMB confines the mass and the radius of PBHs. The mechanism accounting for the decaying process is nothing but PBHs swallowing vacuum energy modes up to a wavelength of the order of their radius. Acting as a natural cut-off on the wavelength of vacuum energy modes, this leads to the observed value for the vacuum energy density.

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One of the most fundamental problems in theoretical physics is the so called *cosmological constant problem*, which is formulated at the interface of cosmology and quantum field theory [1]. An interesting approach towards a possible resolution of this problem is the introduction of a decaying cosmological *term* instead of an evolving one (the so called dark energy). This term, identified with vacuum energy density in Friedmann equations, is taken not to be a constant and decays into other components of the energy-momentum content of the universe namely dark matter or radiation [2, 3]. None of the models based on decaying vacuum energy, presented as yet, are totally satisfactory either due to the lack of a consistent mechanism or because of their inconsistency with the observational data [3] and that is why it is generally being discussed from phenomenological point of view [4].

In the present essay we are going to introduce a consistent mechanism for the above mentioned decay by choosing a physically motivated cut-off frequency for the vacuum energy modes of an effective scalar field. In our scenario it is assumed that the energy-momentum content of the universe consists of three components, matter, radiation and vacuum energy of the above mentioned effective scalar field. Material part is composed of baryonic matter as the visible sector and primordial black holes (PBHs) as the dark sector [5, 6] and the radiation is composed of cosmic microwave background (CMB) photons. Vacuum energy is given by the ground state energy of the effective scalar field modes integrated over all wavelengths from Hubble length as IR cut-off to Planck length as UV cut-off [7, 8]. Planck length as the UV cut-off gives rise to a vacuum energy density which is about 121 orders of magnitude larger than the observed value. Therefore one needs a physically motivated UV cut-off to arrive at the observed value for the vacuum energy density. In our scenario the mechanism inducing such a cut-off is given by the following three (assumptions) steps:

1-There are PBHs in the universe distributed homoge-

neously and isotropically at the large scale.

2-Mass of these black holes is determined by their local thermal equilibrium (LTE) with the CMB.

3-Radius of these PBHs is a natural cut-off wavelength for the vacuum energy density.

In the presence of these PBHs, only those modes of vacuum survive whose wavelengths are larger than the radius of the PBHs and the shorter wavelength modes are eaten up by the PBHs. To examine the feasibility of the above proposed mechanism, an order of magnitude calculations for the vacuum energy density could be carried out as follows : local equilibrium condition between PBHs with temperature $T_{PBH} = \frac{1}{8\pi Gm}$ and radius $r = 2Gm$ on the one hand and CMB with temperature $T_{CMB} = 2.347 \times 10^{-4}\text{eV}$ on the other hand i.e. $T_{PBH} = T_{CMB}$, leads to $r \simeq 339 \text{ eV}^{-1}$. Consequently the radius of PBH determines the cut-off wavelength i.e. $\lambda_c \sim r$ and thereby $k_c = \frac{2\pi}{\lambda_c} = \frac{2\pi}{r} = \frac{\pi}{Gm} \simeq 1.85 \times 10^{-2} \text{ eV}$. Inserting k_c in the expression for vacuum energy density of a massless scalar field we end up with

$$\rho_v = \frac{1}{16\pi^2} k_c^4 \simeq 74.6 \times 10^{-11} \text{ eV}^4 \quad (1)$$

which is fairly close to the observed critical density $\rho_c \simeq 3.96 \times 10^{-11} \text{ eV}^4$ [9]. In what follows, based on the above scenario, we study cosmic evolution of the constituents of the universe. To be more specific a primordial (Schwarzschild) black hole is taken to be in a locally stable thermal equilibrium with its surroundings, if a thermal bath located at $2Gm < r < 3Gm$, encloses the black hole (taking $\hbar = c = 1$). Hawking temperature at r , radial coordinate from the center of the black hole, is given by [10]

$$T(r) = \frac{1}{8\pi Gm} \left(1 - \frac{2Gm}{r} \right)^{-\frac{1}{2}}. \quad (2)$$

Assuming $r \geq 0$ and $T \geq 0$ and provided $rT \geq \frac{\sqrt{27}}{8\pi}$ one can solve the above equation for m , mass of the black

hole, to obtain the following two approximate solutions

$$m_1 \simeq \frac{1}{8\pi GT} \left(1 + \frac{1}{8\pi rT} \right), \quad (3)$$

$$m_2 \simeq \frac{1}{2G} r \left(1 - \frac{1}{(4\pi rT)^2} \right). \quad (4)$$

m_2 being the stable solution [10] is taken to present the PBHs' mass while in equilibrium with CMB as the thermal bath. It is assumed that CMB as a thermal bath, constructs a cavity around the black hole effectively located at some $r = xGm_2$ where $x \in (2, 3)$ [19]. Substituting $r = xGm_2$ in Eq.(4) and solving for T we have

$$T = \frac{1}{4\pi Gm_2 \sqrt{x^2 - 2x}}, \quad (5)$$

where setting T equal to T_{CMB} does not fix m_2 due to the extra degree of freedom x . On the other hand while upper bound of x fixes the lower bound of m_2 ($m_2 = \frac{1}{4\pi G\sqrt{3}T_{\text{CMB}}} \simeq 2.9 \times 10^{58} \text{eV}$), at its lower bound ($x = 2$), m_2 diverges and by the previous argument gives rise to a zero vacuum energy density. Fortunately observations set an upper bound of $10^{26} \text{gr} - 10^{27} \text{gr}$ on the mass of PBHs [11, 12] and in consequence vacuum energy density, although very small, is not zero. Hence adopting the above theoretical lower bound and the observational upper bound, the mass of PBHs is in the range $2.9 \times 10^{58} \text{eV} - 5.6 \times 10^{59} \text{eV}$. Now identifying ρ_Λ with ρ_v and using $\rho_\Lambda = 0.7\rho_c$ for the present value of the vacuum energy density, Eq.(1) along with $k_c = \frac{\pi}{Gm_2}$, set $m_2 \sim 5.7 \times 10^{58} \text{eV}$ for the present value of PBH mass which is, interestingly enough, in the above mass range.

Looking for the contribution of vacuum energy in the cosmic evolution, we substitute $k_c = \frac{\pi}{Gm_2}$ in Eq.(1) to get the following relation between vacuum energy density and the mass of PBHs,

$$\rho_\Lambda = \frac{\pi^2}{16G^4} \frac{1}{m_2^4}, \quad (6)$$

according to Eq.(5), m_2 is a function of T and x [20] and in consequence vacuum energy density would be a function of temperature and consequently a function of time. Hence we need to determine the functional dependence of $m_2(t)$ on cosmic time and to do so we employ the energy conservation law

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (7)$$

where now

$$\begin{aligned} \rho &= \rho_D + \rho_b + \rho_\gamma + \rho_\Lambda, \\ p &= p_D + p_b + p_\gamma + p_\Lambda, \end{aligned} \quad (8)$$

with

$$p_D = p_b = 0, \quad p_\gamma = \frac{1}{3}\rho_\gamma, \quad p_\Lambda = w_\Lambda \rho_\Lambda, \quad (9)$$

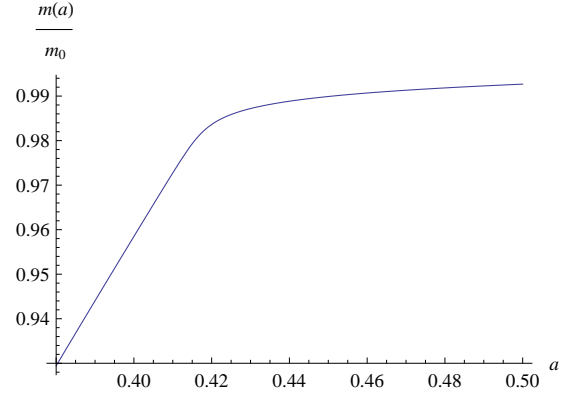


FIG. 1: $\frac{m(a)}{m_0}$ in terms of a for $w_\Lambda = -0.99$.

and in which $\rho_D = mn$ is the dark matter density with n the number density of PBHs and m their mass (hereafter we drop the index of m_2), $\rho_\gamma = \frac{\pi^2}{15}T^4$ is the energy density of CMB (hereafter its temperature is denoted by T), $\rho_b = \rho_{0b}a^{-3}$ is the baryonic energy density (index of zero on any quantity represents the value of that quantity at the present epoch) and $w_\Lambda \simeq -1$. We assume that the total number of PBHs is conserved i.e the conservation equation for n is

$$\dot{n} + 3Hn = 0, \quad (10)$$

implying $n = n_0 a^{-3}$. Substituting Eqs.(8), (9) and (10) in Eq.(7) leads to

$$\dot{m}n + \dot{\rho}_\Lambda + 3(1 + w_\Lambda)H\rho_\Lambda = 0. \quad (11)$$

Taking into account that ρ_Λ is a function of m and by defining $m' = \frac{dm}{da}$, Eq.(11) becomes

$$m' \left(n + \frac{d\rho_\Lambda}{dm} \right) + \frac{3}{a}(1 + w_\Lambda)\rho_\Lambda = 0, \quad (12)$$

in which using Eqs. (10) and (6) we obtain

$$m' \left(n_0 a^{-3} - \frac{\pi^2}{4G^4 m^5} \right) + \frac{3}{a}(1 + w_\Lambda)\rho_\Lambda = 0. \quad (13)$$

This equation gives m as a function of a and consequently a function of cosmic time. Having $m_0 = 5.7 \times 10^{58} \text{eV}$ and $\rho_{D0} = 0.22\rho_c$ results in $n_0 = 1.5 \times 10^{-70} \text{eV}^3$ which is then used to solve Eq.(13) for $m(a)$ numerically, the result of which is plotted in FIG 1 for $w = -0.99$. It should be noted that small deviation of w_Λ from -1 has a vital role in getting the right behavior for $m(a)$ and this could be traced back to small interactions and excitations of the scalar field. Now, having $m(a)$, we could obtain dark matter density, $\rho_D = m(a)n(a)$, and vacuum energy density, Eq.(6), as functions of expansion factor. These are plotted together in FIG.2..

As FIG.2 shows, transition point occurs at $a \simeq 0.55$ which is the same value predicted by the Λ CDM model

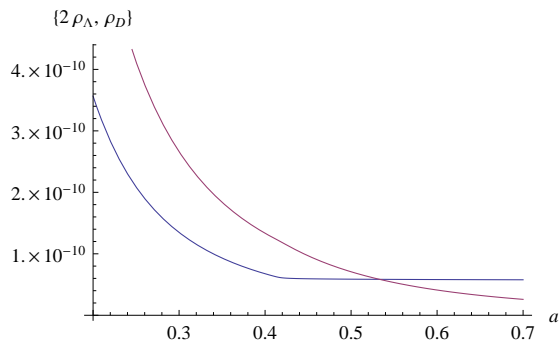


FIG. 2: $2\rho_\Lambda$ (blue) and ρ_D (red) in terms of a .

[13]. Eq.(13) for $w = -1$ and $m' \neq 0$ gives

$$m(a) = \left(\frac{\pi^2}{4G^4 n_0} \right)^{\frac{1}{5}} a^{\frac{3}{5}}, \quad (14)$$

which on using Eqs.(6) and (10) leads to

$$\rho_D = n_0^{\frac{4}{5}} \left(\frac{\pi^2}{4G^4} \right)^{\frac{1}{5}} a^{-\frac{12}{5}}, \quad (15)$$

$$\rho_\Lambda = \frac{1}{4} n_0^{\frac{4}{5}} \left(\frac{\pi^2}{4G^4} \right)^{\frac{1}{5}} a^{-\frac{12}{5}}. \quad (16)$$

Behaviour of ρ_D and ρ_Λ according to the above equations is the same as their behaviour for $a \lesssim 0.4$ in FIG. 2 which was obtained for $w = -0.99$. In other words unlike $m(a)$, for $a \lesssim 0.4$, behaviour of ρ_D and ρ_Λ is not sensitive to small deviations of w from -1 .

Usually $r \equiv \frac{\rho_D}{\rho_\Lambda}$ is used to quantify another well known problem in cosmology, the so called *cosmic coincidence problem*. When r is a constant there is no such a problem but when r is a function of time then we need a mechanism to explain the present value of r . From the above two equations for $a \lesssim 0.4$ we have $r = 4$ whereas, by FIG.2, for $a \gtrsim 0.4$ one obtains the present value of $r_0 \approx 0.30$ [14]. Therefore the above scenario also offers a possible solution to the cosmic coincidence problem and indeed makes the coincidence inevitable.

In our model there are two basic assumptions that need to be clarified, one is the cavity construction around the PBHs by CMB and the other is the accretion of virtual particles of the vacuum onto the black hole. We know that the photons do not have stable orbits around a black hole and according to this fact it seems that cavity construction by CMB is not possible. But it should be taken into account that the photons involved in the discussion of photon orbits have very small wavelengths compared to the characteristic scale in the problem. However in our model, not only the wavelengths of CMB photons are not small compared to the characteristic scale of the system which is here the black hole radius, but also are larger

than that. At the present epoch, according to Wien's displacement law, wavelengths of CMB photons are about 10^{-3} meter while the radius of primordial black holes is about 10^{-4} meter. Hence CMB photons can only scatter elastically from black hole without any energy exchange and because of this effect, the photons are not involved in the conservation law given by Eq.(7).

Accretion of virtual particles of vacuum onto a black hole is a controversial issue. According to [15, 16] it is a possible effect. But others, referring to the study carried out in [17], do not accept this possibility. Indeed one can not rely on the results obtained in [17] on this matter, because that study is essentially based on the classical theory of black holes whereas the mentioned accretion process is a quantum mechanical effect in nature. As Hawking radiation makes it clear, quantum effects of vacuum give rise to the radiation of *static* black holes [18], while the classical theory has nothing to say about radiation from these black holes.

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* Electronic address: zeynizadeh@physics.sharif.ir

† Electronic address: nouri@khayam.ut.ac.ir

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